Innovation And Imitation At Various Stages Of Development: A Model With Capital

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Abstract

A simple model of technological imitation and innovation is developed to explain the club convergence of countries' per-capita GDP. It is shown that stable regimes may be based on imitation, innovation or mixed policy. Using World Bank and ICRG data, we estimate how innovation and imitation costs depend on the relative productivity, savings rate and institutional quality. These results allow to adjust the parameters of the model, so that it gives qualitatively correct picture of the evolution of countries and club convergence. Taking into account capital accumulation does not affect the structure of equilibria, but improves the precision of estimation.

1 Introduction

A complicated picture of relative productivity growth in different countries is one of the main challenges for the modern economic science. Advanced economies seem to be converging to each other. Another converging group includes a number of Latin American and some other countries with 20-30% of the USA GDP per capita. These two groups seem to be growing with equal rates whereas most of African countries fall behind. It looks like world income is moving toward a distribution with two peaks. This observation, made by Quah (1993), gave birth to a number of research about "club convergence".

In fact, some explanations of this phenomenon may be got in framework of underdevelopment or poverty trap theories. There are four different classes of development trap models that consider the trap as a result of a lack of physical capital, human capital, productivity, and low quality of economic and political institutions (see Azariadis and Drazen (1990) and Feyrer (2003) for a survey and references).

Easterly, Levine (2000), and Feyrer (2003) found, however, that factor accumulation can not explain two peaks distribution: "the output per capita is tending toward twin peaks despite the tendency toward convergence in the accumulable factors. The productivity residual, on the other hand, shows movement similar to the distribution of per capita output..." (Feyrer (2003), p.31). Thus interactions between innovation and imitation processes and institutional development should play dominant roles in modelling of the club convergence behavior.

Up to the end of eighties, innovation and imitation, two sides of the development process, were modelled separately. Segerstrom (1991) cites the only exception: a paper by Baldwin and Childs (1969) where firms can choose between innovating and imitating. Iwai (1984a,b) considered distributions of firms in an industry with respect to efficiency and suggested a model that describes evolution of the distribution curves. The model demonstrates a "dynamic equilibrium" between innovation and imitation processes. This approach, in principle, may be implemented to country distributions as well. It is developed in a number of papers by Henkin and Polterovich (see Polterovich, Henkin (1988) and Henkin, Polterovich (1999) for a survey). They assume that a firm moves to the neighboring efficiency level due to innovation and imitation of more advanced technologies, and the speed of the movement depends on the share of more advanced firms. The movement is described by a difference-differential equation that may be considered as an analogue of the famous Burgers partial differential equation. It is shown that innovation-imitation forces may split an isolated industry into several groups so that the solution looks like a combination of several isolated waves moving with different speeds.

Segerstrom (1991) develops a dynamic general equilibrium model and studies its steady state in which some firms devote resources to discovering new products and other firms copy them. Somewhat similar model is suggested in Barro and Sala-I-Martin (1995) to describe the interaction between innovation and imitation as an engine of economic growth. They show that follower countries tend to catch up to the leader because imitation is cheaper than innovation. Barro and Sala-I-Martin discuss also a number of related issues and papers devoted to technological diffusion, R&D races and leapfrogging. The theory of leapfrogging is developed in Brezis, Krugman, and Tsiddon (1993).

In a recent paper by Acemoglu, Aghion, and Zilibotti (2002), it is shown that countries at early stages of development use imitation strategy whereas more advanced economies switch to innovation-based policy. Relatively backward economies may switch out of investment-based strategy too soon or fail to switch at all. In the latter case, they get into a non-convergence trap. The traps are more likely if the domestic credit market is imperfect. The authors receive also a number of other results that characterize policies for different stages of development.

In their paper, Acemoglu, Aghion, and Zilibotti implicitly assume that all countries imitate the most advanced technology and that there is the "advantage of backwardness": relatively backward countries have the same opportunities to imitate as more developed ones and experience faster technological development due to the imitation. Such an assumption does not take into account the problems of adapting modern technologies. This problems seem to be more serious for wider technology gap. Howitt and Mayer-Foulkes (2002) include this effect in their model of convergence clubs using Shumpeterian dynamics. In their model, the Poisson arrival rate of a technological improvement negatively depends on the country's extent of backwardness. This dependency may cause non-convergence. However, in that model, underdevelopment traps and different convergence clubs occur only when the authors introduce two levels of innovation/imitation intensity, what they call "implementation" and "modern R&D", with the latter option accessible only for developed economies.

Our model (its first version was suggested in Polterovich, Tonis (2003)) borrows a number of elements from Acemoglu, Aghion, and Zilibotti (2002). We concentrate on the innovation-imitation tradeoff and develop this part of their model.

Our approach is different from approaches used by other authors in several important aspects.

1. We do not assume, as it is done in other papers, that a country always imitates the most advanced technology. This assumption seems to be too strict. Every developing country experiences a lot of failures connected with attempts of borrowing the most recent achievements of the developed world. It is too often that modern techniques and technologies turn out to be incompatible with domestic culture, institutions, quality of human capital, or domestic technological structure.

A rational policy admits borrowing of the past experience of the leaders. Imitation of a less advanced technology is cheaper and has more chances for a success.

2. We make difference between global and local innovations. The first ones may be borrowed by other countries whereas the second ones are country specific. It is quite plausible that the most part of R&D expenditures are spent for local innovations[1]. If one likes to use R&D expenditures as a measure for the innovation activity, one should not assume that all innovations produced are useful for other countries[2].

3. The larger is an innovation or an imitation, the less is the probability to produce it (in accordance with Howitt and Mayer-Foulkes, 2002). Therefore, both policies exhibit decreasing rate of return. The tradeoff between innovation and imitation projects may result in producing both of them in positive proportions.

4. Costs of imitation and innovation in a country depend on its relative level of development. The level of development is defined as a ratio of the country productivity parameter to the productivity of the most advanced economy. A similar assumption is used in Acemoglu, Aghion, and Zilibotti (2002), where the costs per unit of the productivity increase are taken to be constant for imitation and linear for innovation. We consider quite general shape of cost functions. It is assumed, however, that the value of imitation cost function increases and the value of innovation cost function decreases when the economy approaches a leader. A similar assumption about the cost of imitation may be found in Barro and Sala-I-Martin (1995). The reason is that less advanced economies may borrow well-known and cheap technologies that may be even obsolete for advanced economies.

   The shape of dependence of unit innovation cost on development levels is more questionable. On the one hand, one could assume decreasing rate of return, on the other hand, accelerating effects of the technical progress occur. We analyze empirical data to demonstrate that unit innovation cost is probably less for most advanced economies.

5. We take into account the country's institutional quality that affects imitation and innovation cost functions. Since the institutional indicators are considered as exogenous, we study the behavior of the economy for a broad class of expenditures functions.

[1] In Pack (2001, p.114) the following passage from Ruttan (1997) is cited: "The technologies that are capable of becoming the most productive sources of growth are often location specific". Pack writes that "this view is widely shared among those who have done considerable research on the microeconomics of technology".

[2] In Russia in 2001, R&D expenditures amounted to 100.5 billion rubles, whereas the value of exported technologies was only 19 billion rubles. (Russian Statistical year book, 2002. M.: Goskomstat, pp. 521–522). There were granted more than 16000 patents, and only 4 of those were exported.

6. In most of investigations, steady states are studied only. However, to generate a picture that would resemble a real set of country trajectories, one has to consider transition paths. To do that, we are forced to simplify our model drastically. The model is quasi-static and generates trajectories as sequences of static equilibria[3]. We make use of many other simplifications as well.

Many authors, who study endogenous growth models, do not consider capital accumulation (see Aghion and Howitt, 1998, or Barro and Sala-I-Martin, 1995). In Polterovich, Tonis (2003) we use this simplification as well. It was shown that in the simple model suggested, there are three types of stationary states, where only imitation, only innovation or a mixed policy prevails. It was demonstrated how one can find the stationary states and check their stability for a broad class of imitation-innovation cost functions.

Using World Bank and International Country Risk Guide (ICRG) statistical data for the period of 1980-1999, we revealed the dependence of innovation and imitation cost on GDP per capita measured in PPP and on indicator of investment risk. An appropriate choice of five adjustment parameters of the model gave a possibility to generate trajectories of more than 80 countries and, for most of them, got quantitatively correct pictures of their movement. We found that the model generates a picture qualitatively similar to the real one. Roughly speaking, three groups of countries behave differently. There is a tendency to converge within each group. Countries with low institutional quality have stable underdevelopment traps near the imitation area. Increase in the quality moves the steady state toward a better position and turns into a new stable steady state where local innovations and imitations are jointly used. Under further institutional improvements, a combined imitation-innovation underdevelopment trap disappears. All countries with high quality of institutions are moving toward the area where pure innovation policy prevails.

It was mentioned above that, in accordance to empirical findings, capital accumulation does not play a decisive role in the formation of clubs. However the conclusion is related to long run behavior only. It is important to know to what extent the picture is changing if one takes capital into account. Below we suggest and study a modification of our previous model, this modification describes capital accumulation as well[4]. We show that, under reasonable assumptions, the choice between innovation and imitation activities does not depend on the stock of the capital accumulated in an economy. The choice is determined by the relative innovation and imitation costs and productivities that depend on the relative technological level of a country and, maybe, on some exogenous parameters including

[3]This is a feature of the Acemoglu, Aghion, and Zilibotti's model as well.

[4]An attempt to include capital to the two-sector modification of Polterovich, Tonis (2003) model was made by A. Semenov in his MA thesis (Semenov, 2004). In this model, only pure innovation or pure imitation equilibria exist. The model does not aim to explain the club convergence.

institutional qualities and savings rates. Due to this fact the structure of asymptotic behavior remains the same as in the model without capital. Thus the model explains the empirical observation. A calibration of the model reveals, however, that the asymptotic behavior may depend on the saving rate (it is considered as an exogenous parameter of the model). We show also that taking into account the evolution of the capital stock improves the quality of approximation of the real data.

2 Description of the model

Consider a multi-product economy evolving over time. There are three kinds of goods in this economy: final good, capital and the continuum set of high-technology intermediate goods indexed by $\nu \in [0,1]$. Every period, final good is competitively produced from the intermediate goods. Each intermediate good ν is characterized by its productivity A_ν. The production function for the final good is given by

$$Y = \int_0^1 F(A_\nu, X_\nu) d\nu, \tag{1}$$

where X_ν is the quantity of intermediate good ν involved in the production process and F is homogeneous of degree 1:

$$F(A_\nu, X_\nu) = A_\nu f(x_\nu), \qquad x_\nu = \frac{X_\nu}{A_\nu}. \tag{2}$$

(x_ν is a "normalized" quantity of good ν). Here we assume that f satisfies Inada conditions and $xf''(x) + f'(x)$ falls from ∞ to 0, as x proceeds from 0 to ∞. Through the paper, a special case of a Cobb — Douglas production function will be considered: $f(x) = x^\alpha$ $(0 < \alpha < 1)$, in which case $F(A, X) = A^{1-\alpha} X^\alpha$.

We consider that final good can be sold in the world market at price 1. The price of intermediate good ν is p_ν. The producer of final good chooses its demand for each of intermediate goods X_ν, taking all prices as given, so as to maximize its profit:

$$Y - \int_0^1 p_\nu X_\nu d\nu \to \max. \tag{3}$$

Intermediate goods can be produced from capital. The production function in the intermediate goods sector is assumed to be linear: one unit of capital can be converted to one unit of intermediate good[5]. In each sector $\nu \in [0,1]$, only one firm enjoys the full access to the technology of producing

[5] We can use one-to one production function because the unit of intermediate good ν along with the technological parameter A_ν can be properly adjusted

the corresponding intermediate good, so the market for each intermediate good is monopolistic. Let the rental price of capital be r. Then firm's profit V_ν is given by

$$V_\nu = (p_\nu - r)X_\nu. \tag{4}$$

The firm is facing the demand of the final good sector for its product and monopolistically chooses p_ν so as to maximizes its profit V_ν.

In each sector ν, the monopolist firm lives for one period of time[6]. At the beginning of the period, all sectors starts with the same (country-specific) productivity level A. This level represents the cumulative technological knowledge achieved by the economy up to that date. Prior to producing its intermediate good, each firm may perform technological innovations and imitations, thus raising its productivity from A to A_ν, and then produces input with productivity A_ν (it will be described later on, how A_ν is determined).

The above considerations concern the domestic economy. There are also foreign countries, in which initial productivity levels may differ from that of the domestic country. Denote by $\bar{A}$ the initial productivity level of the most developed economy. Along with the domestic absolute productivity level A, let us consider the relative level $a = \dfrac{A}{\bar{A}}$ which measures the distance to the world technology frontier. It represents the position of the domestic technologies among other ones.

Now let us describe the evolution of technologies. As we said, each firm performs imitation and/or innovation prior to production. Let b_1 and b_2 denote, respectively, the size of the imitation and innovation project. Each project may result in one of two outcomes, success or failure. If the imitation (innovation) was successful, firm's productivity rises at growth rate b_1 (b_2); otherwise, it remains the same. If both projects were successful, the productivity grows at rate $b_1 + b_2$. Thus, after both actions, the technology variable $A(\nu)$ is given by

$$A_\nu = (1 + \xi_{1\nu})(1 + \xi_{2\nu})A, \tag{5}$$

where $\xi_{1\nu}$ ($\xi_{2\nu}$) is a random variable equal to b_1 (b_2) in the case of successful imitation (innovation) by firm ν and 0 otherwiseThis multiplicative probability function brings about a complementarity effect between imitation and innovation: a progress in opportunities for imitation results in more innovation and vice versa.

Firms cannot imitate technologies which have not been developed anywhere in the world yet, so the size of the imitation project is subject to constraint

$$(1 + b_1) \leq \frac{1}{a}, \tag{6}$$

[6] Unlike the model by Acemoglu, Aghion, and Zilibotti (2002a), in which firms live for two periods and form overlapping generations.

where $a = \frac{A}{\bar{A}}$. The maximal productivity level $\bar{A}$ is generated by the leader economy and is not affected by innovations made by less advanced economies. Thus, we assume that the followers produce local innovations which are country-specific and cannot be imitated. We assume that the probabilities of success are, respectively, $\psi_1(b_1)$ and $\psi_2(b_2)$, where $\psi_i(b_i)$ is decreasing in b_i, $\psi_i(0) = 1$ and $b_i\psi_i(b_i)$, the expected value of the corresponding technological growth rate, is bounded from above ($i = 1, 2$). In particular, we consider the following special case of $\psi_i(b_i)$:

$$\psi_i(b_i) = \frac{\mu_i}{\mu_i + b_i}, \tag{7}$$

where $\mu_i > 0$ is a parameter. The expected productivity growth rate that the project can yield is then $\frac{\mu_i b_i}{\mu_i + b_i}$, which cannot be higher than μ_i, so μ_i may be treated as a natural ceiling for possible growth rates due to the corresponding activity (imitation or innovation).

Now let us introduce the costs of imitation and innovation. Technological development includes not only invention or adoption of new methods of production, but also implementation of these methods to the existent machinery. To adopt the costs of spreading technological knowledge over the economy, we assume that the costs of imitation and innovation depend not only on the size of the corresponding project b_i, but also on the amount of capital to be upgraded. Specifically, in order to undertake project b_i, the firm has to invest Kq_ib_i units of capital, where K is the average capital stock over the economy.

The distance to the world technology frontier may also affect the growth opportunities. So, it is supposed that q_1 and q_2, the per-unit costs of imitation and innovation, are not constant, they depend on the relative average productivity level of our economy at the beginning of the period[7]: $q_i = q_i(a)$ (q_i are continuous and differentiable in a). We assume (and this is supported by the empirical evidence) that $q_1(a)$ is increasing and $q_2(a)$ is decreasing in a. According to this assumption, it gets more difficult to imitate and easier to innovate, as the domestic technology gets closer to the world technology frontier. The reasons may be the following. On the one hand, less developed countries must imitate less advanced technology to grow by 1% and it is typical that less advanced technologies are less protected by intellectual property laws; they are also easy to implement because of a lot of experience accumulated by other imitators. On the other hand, the innovation process is likely to exhibit some economy of scale due to a positive externality exerted by the stock of almost accumulated knowledge, so more advance countries incur less costs.

Thus, both forms of technological development are modelled in a similar way here. However, the opportunities for imitation and innovation change in different ways as a increases. In particular, when

[7]Generally, q_1 and q_2 may depend also on some other country-specific parameters. In the empirical section, we consider q_i as functions of savings rate and an indicator of institutional quality.

a is close to 1, there is almost nothing to imitate, whereas innovation is possible and its cost is low.

Under the above assumptions, the expected profit of the firm (net of technology investment expenditures) is given by

$$E(\Pi_\nu) = E(V_\nu) - rZ_\nu, \tag{8}$$

where V_ν is given by the profit maximization in (4), the expectation is taken over the four possible realizations of success/failure of innovation/imitation indicated by random variables $\xi_{1\nu}$ and $\xi_{2\nu}$ and Z_ν is the amount of capital invested in innovation and imitation:

$$Z_\nu = K(q_1(a)b_1 + q_2(a)b_2). \tag{9}$$

In the beginning of the period, the firm chooses b_1 and b_2 so as to maximize its expected net profit given by (8).

The technological parameter A changes from period to period. We assume that each sector ν produces in the next period something slightly different from current-period output, so the current technology cannot be directly used in the next period. Thus, technological knowledge depreciates (gets obsolete) to some extent. In accordance with this assumption, we define the next-period initial productivity level A_{+1} as follows:

$$A_{+1} = (1-\rho)\tilde{A}, \tag{10}$$

where

$$\tilde{A} = (1+\psi_1(b_1)b_1)(1+\psi_2(b_2)b_2)A \tag{11}$$

is the average of the achieved productivity level A_ν over the economy[8] and $0 \leq \rho \leq 1$ is the depreciation rate ($\rho = 0$, if the intertemporal technology transfer is costless, i. e. next-period intermediate goods are the same as current-period ones; $\rho = 1$, if, on the contrary, the current-period technology is absolutely useless for the production in the next period). Here A_{+1} plays the same role for the next-period firms as A for the current-period firms. Note that in the next period (as well as in the current one), all firms start from the same initial productivity, so there is total spillover of technological knowledge among sectors. The assumption about independence of industry's future productivity level on its current innovation/imitation efforts seems to be very restrictive (in particular, the model does not describe the behavior of long-run investors); it is imposed only for the sake of simplicity.

To finish with the description of the model, we need to specify, how capital evolves over time and how its price is formed. Let K and K_{+1} be the capital stocks at the current period and the next

[8] Note that despite all A_ν are stochastic, their average $\tilde{A}$ is non-random because the set of sectors is continual.

period, respectively. We assume that the next-period capital stock is determined by

$$K_{+1} = (1-\delta)K + \sigma Y, \tag{12}$$

where $\delta \in [0,1]$ is the (physical) depreciation rate, Y is the total output of the final good and $\sigma \in [0,1]$ is the (constant) saving rate. Parameter σ is assumed to be country-specific. It may depend on the quality of institutions and investment climate in the country. We also assume that $\rho < \delta$, i. e. physical capital depreciates faster than technology.

The capital market is perfectly competitive. The equilibrium rental rate r is determined by the balance equation:

$$\int_0^1 (X_\nu + Z_\nu)d\nu = K. \tag{13}$$

A dynamic equilibrium in the model is defined as the set of variables $\{K, A, X, b_1, b_2, r, p\}_t$ (in each period t), such that all parties make their optimal decisions concerning X, b_1 and b_2, with evolution and balance equations (10), (12), (13) holing in each period.

To summarize, the economy evolves as follows. At the beginning of the period, all leader firms start from the same productivity level A. Firms choose the sizes of their imitation and innovation projects which maximize their profits. Then random events are realized: success or failure of these projects (correspondingly, random variables A_ν are evaluated). Then production takes place and profits are earned. The next-period productivity level A_{+1} is then determined by (10). The next-period capital stock is determined by (12). All next-period firms start their projects from productivity level A_{+1}. They make their innovations and imitations, thus determining A_{+2}, their successors' productivity level, and so on.

3 Analysis of the model

In this section, imitation, innovation and production strategies in each country are obtained and the dynamics or relative growth is studied based on the model described above.

Let us start with determining the demand for intermediate good. Since the final good sector is competitive, the final good producers' demand for intermediate good ν is given by

$$p_\nu = f'(x_\nu). \tag{14}$$

The monopolist in sector ν chooses pair (x_ν, p_ν) subject to (14) so as to maximize its profit (4). Profit maximization yields the optimal quantity of intermediate good produced by the monopolist:

$$X_\nu = A_\nu x,$$

where x is a solution to

$$r = xf''(x) + f'(x) \tag{15}$$

(due to our assumptions, this solution exists and is unique). So, x_ν will be the same in all sectors. Due to (14), all prices p_ν will be the same too.

The corresponding expected monopolistic profit (not taking into account the costs of innovation/imitation) is given by

$$E(V_\nu) = e(x)\tilde{A} = e(x)v(b_1, b_2)A,$$

where $e(x) = -x^2 f''(x)$, $v(b_1, b_2) = v_1(b_1)v_2(b_2)$, $v_i(b_i) = (1 + \psi_i(b_i)b_i)$.

Let us denote by $k = \dfrac{K}{A}$ the capital stock normalized by productivity. According to (8), the expected net profit is given by

$$E(\Pi_\nu) = A(e(x)v(b_1, b_2) - r(x)k(q_1(a)b_1 + q_2(a)b_2)), \tag{16}$$

where $r(x) = xf''(x) + f'(x)$ (see (15)).

The firm maximizes the right-hand side of (16) subject to constraints $0 \le b_1 \le \bar{b}_1(a)$ and $b_2 \ge 0$, where $\bar{b}_1(a) = \dfrac{1}{a} - 1$ is the maximal feasible level of imitation (according to (6)). It is convenient for representing the solution to this problem to introduce the following notations:

$$h(x) = \frac{e(x)}{r(x)} = -\frac{x^2 f''(x)}{f'(x) + xf''(x)}; \tag{17}$$

$$\varphi_i(b_i) = \frac{\partial v}{\partial b_i} = \psi_i(b_i) + b_i \psi_i'(b_i) \qquad (i = 1, 2). \tag{18}$$

Note that due to our assumptions, $h(x)$ is increasing in x and $\varphi_i(b_i)$ is decreasing in b_i so that $\varphi_i(0) = 1$ and $\varphi_i(\infty) = 0$. It is easy to see that in the Cobb-Douglas case ($f(x) = x^\alpha$), $h(x)$ is a linear function: $h(x) = \eta x$, where $\eta = \dfrac{1}{\alpha} - 1$.

Using these notations, we can easily obtain the optimal innovation/imitation strategy chosen by the firm:

Lemma 1 *Suppose that for given pair (k, a), production activity x is chosen optimally by all firms in the economy. Then the optimal b_1 and b_2 are determined by the following equation system:*

$$\begin{aligned} b_1 &= \tilde{b}_1(k, x, a) = \min\left(\max\left(\varphi_1^{-1}\left(\frac{kq_1(a)}{v_2(b_2)h(x)}\right), 0\right), \bar{b}_1(a)\right); \\ b_2 &= \tilde{b}_2(k, x, a) = \max\left(\varphi_2^{-1}\left(\frac{kq_2(a)}{v_1(b_1)h(x)}\right), 0\right). \end{aligned} \tag{19}$$

Proof. Provided that the solution is interior, i. e. no constraint is binding, the first-order conditions for the optimal choice of b_i are the following:

$$v_j(b_j)h(x)\varphi_i(b_i) = kq_i(a) \qquad (i = 1, 2;\ j \neq i). \tag{20}$$

Adopting all constraints, we obtain (19). ■

Thus, due to (19), b_1 and b_2 can be determined as far as k, x and a are given. In equilibrium, x can be determined by k and a from the balance equation (13), which may be rewritten as

$$k = v(b_1, b_2)x + z(b_1, b_2)h(x), \tag{21}$$

where

$$z(b_1, b_2) = \frac{Z}{Ah(x)} = v_2(b_2)b_1\varphi_1(b_1) + v_1(b_1)b_2\varphi_2(b_2) \tag{22}$$

and b_1 and b_2 are given by (19). So, x, b_1 and b_2 can be represented as functions of two phase variables, k and a: $x = \hat{x}(k, a)$, $b_i = \hat{b}_i(k, a)$.

Lemma 2 *In the Cobb-Douglas case ($f(x) = x^\alpha$), $\hat{b}_1(k, a)$ and $\hat{b}_2(k, a)$ actually depend only on a and are independent of k. Given a, pair $(\hat{b}_1(a), \hat{b}_2(a))$ can be determined from the following equation/inequality system:*

$$\begin{aligned} q_1(a) &\underset{(\geq)}{=} \frac{\eta v_2(b_2)\varphi_1(b_1)}{v(b_1, b_2) + \eta z(b_1, b_2)}; \\ q_2(a) &\underset{(\geq)}{=} \frac{\eta v_1(b_1)\varphi_2(b_2)}{v(b_1, b_2) + \eta z(b_1, b_2)} \end{aligned} \tag{23}$$

(*inequality holds, if the corresponding* $b_i = 0$).

Proof. In the Cobb-Douglas case, equation (21) takes the form

$$k = (v + \eta z)x \tag{24}$$

and the first-order conditions for the interior solution (20) take the form

$$\eta v_j(b_j)\varphi_i(b_i) = (v + \eta z)q_i(a) \qquad (i = 1, 2;\ j \neq i). \tag{25}$$

Since v and z are functions of b_1 and b_2 and do not depend neither on x, nor on k, then (25) implies that b_1 and b_2 depend only on a. Taking into account boundary solutions (with $b_1 = 0$ or $b_2 = 0$), we obtain (23). ■

Lemma 2 shows that the process of technological development is independent of the stock of accumulated capital (normalized by productivity) under Cobb-Douglas production function. So, the impact of capital on GDP growth may reveal only during the transition period, not in the long run.

So far, we have not taken into account that constraint (6) can be binding (this is the case, when solution b_1 of system (23) is higher than $\bar{b}_1(a)$). Now we are going to adopt this possibility.

Lemma 3 *Suppose that $f(x) = x^\alpha$ and consider an equilibrium, at which* (6) *is binding in the current period. Then $\hat{b}_1(a) = \bar{b}_1(a)$ and $\hat{b}_2(a) = \bar{b}_2(a)$, where $\bar{b}_2(a)$ is a solution to*

$$1 - q_1(a)\bar{b}_1(a) - q_2(a)b_2 \underset{(\geq)}{=} \frac{v_2(b_2)q_2(a)}{\eta\varphi_2(b_2)} \tag{26}$$

with respect to b_2, where the inequality holds, if $b_2 = 0$.

Proof. If (6) is binding, then $b_1 = \bar{b}_1(a)$. The expression for the normalized capital investment in innovation and imitation takes the form

$$\tilde{z} = \frac{Z}{Ah(x)} = \frac{q_1(a)\bar{b}_1(a)k}{\eta x} + v_1(\bar{b}_1(a))b_2\varphi_2(b_2). \tag{27}$$

Combining (27) and (24), with $\tilde{z}$ standing for z, we obtain the following relationship between x and k:

$$x = \frac{(1 - q_1(a)\bar{b}_1(a))k}{v_1(\bar{b}_1(a))(v_2(b_2) + \eta b_2\phi_2(b_2))}. \tag{28}$$

Hence, b_2 is determined by (26). ■

Let us denote $\tilde{q}_1(a)$ and $\tilde{q}_2(a)$ as follows:

$$\tilde{q}_i(a) = \frac{\eta v_j(b_j)\varphi_i(b_i)}{v(b_1, b_2) + \eta z(b_1, b_2)} \quad (j \neq i), \tag{29}$$

if constraint (6) is binding and $\tilde{q}_i(a) = q_i(a)$, otherwise (v and z are given by (22), as usual). Then b_1 and b_2 are in any case determined by system (23), with $\tilde{q}_i(a)$ standing for $q_i(a)$, no matter, whether constraint (6) is binding or not. It is convenient to consider these "adjusted" per-unit cost functions $\tilde{q}_i(a)$ instead of $q_i(a)$, because they allow to ignore constraint (6).

All the above results can be summarized in the following proposition:

Proposition 1 *Suppose that $f(x) = x^\alpha$. Then for a country with given country-specific exogenous parameters, the equilibrium levels of innovation and imitation activity depend only on the pair of adjusted per-unit costs $\tilde{q}(a) = (\tilde{q}_1(a), \tilde{q}_2(a))$ (see Figure 1 on page 19):*

1. *If $\tilde{q}(a) \in D_{00}$, then the economy stagnates with no technological development;*
2. *If $\tilde{q}(a) \in D_{+0}$, then there is only imitation and no innovation;*
3. *If $\tilde{q}(a) \in D_{0+}$, then there is only innovation and no imitation;*
4. *If $\tilde{q}(a) \in D_{++}$, then a mixed policy including innovation and imitation is used.*

As it was noted before, the multiplicative form of the probability function yields some complementarity between imitation and innovation. However, the competition in the capital market may bring about the opposite effect. It turns out that if b_1 and b_2 are not very high, then the total effect is substitutionary. Indeed, provided the interior solution ($b_i > 0$) and the Cobb-Douglas form of the production function, b_1 can be determined from

$$q_1 = \frac{1}{b_1 + \dfrac{v_1(b_1)}{\varphi_1(b_1)}\left(\dfrac{1}{\eta} + \dfrac{b_2\varphi_2(b_2)}{v_2(b_2)}\right)} \tag{30}$$

Suppose that q_2 gets higher (by some reason) and q_1 remains the same. Then, as follows from (30), b_1 rises, if $\dfrac{b_2\varphi_2(b_2)}{v_2(b_2)}$ is increasing in b_2, which is true, if b_2 is not very high. Thus, the substitutability due to the capital balance prevails over the complementarity effect in this case.

Another corollary from the optimality conditions is that b_1 is decreasing and b_2 is increasing in a (again, for not very high b_1 and b_2), i. e., imitation is gradually replaced by innovation, as the economy gets more developed.

Let us study the dynamics for countries with given σ and other country-specific parameters. It is convenient to describe the position of the system by pair of phase variables (k, a). The equation determining the evolution of capital (12) may be rewritten in terms of (k, a) as

$$\Delta k = \frac{\sigma f(x) - (g+\delta)k}{(1-\rho)v}, \tag{31}$$

where $\Delta k = k_{+1} - k$ is the increment in k after the period, $g = (1-\rho)v - 1$ is the growth rate of the domestic productivity. The evolution of the relative productivity level a is given by

$$\Delta a = a_{+1} - a = \left(\frac{v}{\bar{v}} - 1\right), \tag{32}$$

where $\bar{v} = \dfrac{1+\bar{g}}{1-\rho}$ and $\bar{g}$ is the growth rate of the world technology frontier. Here we assume that $\bar{v} > 1$, i. e. the leader undertakes some innovation in equilibrium (the leader must develop technologies itself, because it has nothing to imitate).

A dynamic equilibrium, at which k and a are not changing over time (i. e., $\Delta k = 0$ and $\Delta a = 0$) is called *stationary*. An important question is whether a given stationary equilibrium is stable or not. If it is stable, then convergence takes place within the corresponding group of countries. Otherwise, the equilibrium marks the boundary between the attraction areas of different centers of convergence.

The following proposition yields stability conditions for stationary equilibria.

Proposition 2 *Suppose that $f(x) = x^\alpha$ and consider a stationary equilibrium, at which (6) is not binding. Then*

1. *If there is no innovation in equilibrium, then the equilibrium is asymptotically stable.*

2. *If there is no imitation in equilibrium, then the equilibrium is unstable.*

3. *If both imitation and innovation are present in the equilibrium, then its stability is determined by the value of Γ (see formula (39) below): if $\Gamma > 0$, then the stationary equilibrium is unstable; otherwise, it is stable.*

Proof. Consider an infinitesimal change in (k, a). How will it affect the subsequent evolution? Differentiating the balance equation (21) yields

$$dk = (v + zh'(x))dx + x\,dv + h(x)dz, \tag{33}$$

where dv and dz are given by

$$dv = \frac{\beta h'(x)dx + \gamma k\,da}{h(x)} - \frac{\beta dk}{k}, \tag{34}$$

$$dz = \frac{\tilde{\beta} h'(x)dx + \tilde{\gamma} k\,da}{h(x)} - \frac{\tilde{\beta} dk}{k} \tag{35}$$

and β, γ, $\tilde{\beta}$ and $\tilde{\gamma}$ are defined as follows:

$$\begin{aligned}
\beta &= -\frac{\varphi_1(b_1)^2 v_2(b_2)}{\varphi_1'(b_1)} - \frac{\varphi_2(b_2)^2 v_1(b_1)}{\varphi_2'(b2)};\\
\gamma &= \frac{\varphi_1(b_1) q_1'(a)}{\varphi_1'(b_1)} + \frac{\varphi_2(b2) q_2'(a)}{\varphi_2'(b2)};\\
\tilde{\beta} &= -\frac{(\chi_1(b_1)v_2(b_2) + b_2\varphi_1(b_1)\varphi_2(b_2))\varphi_1(b_1)}{\varphi_1'(b_1)}\\
&\quad -\frac{(\chi_2(b_2)v_1(b_1) + b_1\varphi_1(b_1)\varphi_2(b_2))\varphi_2(b_2)}{\varphi_2'(b_2)};\\
\tilde{\gamma} &= -\frac{(\chi_1(b_1)v_2(b_2) + b_2\varphi_1(b_1)\varphi_2(b_2))q_1'(a)}{\varphi_1'(b_1)v_2(b_2)}\\
&\quad -\frac{(\chi_2(b_2)v_1(b_1) + b_1\varphi_1(b_1)\varphi_2(b_2))q_2'(a)}{\varphi_2'(b_2)v_1(b_1)}.
\end{aligned} \tag{36}$$

Note that β is always positive, whereas γ, $\tilde{\beta}$ and $\tilde{\gamma}$ may be negative or positive, depending on the behavior of the cost functions q_1 and q_2.

Combining (33)–(35), one can solve for dx as a linear function of dk and da. In particular, in the Cobb-Douglas case the formulas for dx and dv take the following simple form:

$$dx = \frac{x}{k}\,dk - \frac{\dfrac{\gamma}{\eta} - \tilde{\gamma}}{v + \dfrac{\beta}{\alpha}}\,da; \tag{37}$$

$$dv = \frac{\Gamma}{\left(v + \frac{\beta}{\alpha}\right) x} da, \tag{38}$$

where

$$\Gamma = \frac{v}{\eta}\gamma + \beta(\gamma - \tilde{\gamma}) = \left(\frac{v\varphi_1(b_1)}{\eta\varphi_1'(b_1)} - \beta b_1\right) q_1'(a) + \left(\frac{v\varphi_2(b_2)}{\eta\varphi_2'(b_2)} - \beta b_2\right) q_2'(a). \tag{39}$$

As follows from (38) and (39), v positively depends on a, if $\Gamma > 0$, i. e. if the cost of innovation falls relatively faster than the cost of imitation rises, as a increases. This is a sign of divergence: different countries may eventually grow at different rates, moving away from each other and from the leader too. In the opposite case, convergence will take place. If the sign of Γ is positive for some a and negative for other, i. e. v is not monotone in a, then the club convergence may take place, with clubs forming around the steady states at which $\Gamma < 0$.

In order to analyze qualitatively the dynamics of the system, let us consider the neighborhood of some (interior) stationary equilibrium (k^*, a^*). Suppose that the phase point (k, a) lies somewhere near (k^*, a^*) and $\Delta a = 0$ at this point (as well as at (k^*, a^*), by definition). Is then Δk negative or positive? Using (34)–(31), one can obtain in the Cobb-Douglas case the following relationships between the increment in Δk, x ,a and v:

$$d(\Delta k) = -\frac{(1-\alpha)\sigma x^{\alpha}}{(1-\rho)k} dk + c\, da, \tag{40}$$

where c is some expression, not depending on any differentials. Due to (40), $d(\Delta k)$ is decreasing in dx, as far as $a = \text{const}$ (or, equivalently, $v = \text{const}$). Hence, the stability of an interior steady state is fully determined by the sign of Γ: if $\Gamma > 0$, then the stationary point is unstable and the system exhibits saddle dynamics around it. Otherwise, the phase curves form a stable knot with faster convergence by k and slower by a.

All the above considerations are valid for the case of interior solution, where $b_i > 0$ and $b_1 < \bar{b}_1(a)$. If $b_1 = 0$, then all the above formulas can be applied if we put $q_1'(a) = 0$ everywhere. In particular, Γ is definitely positive in this case, so all countries diverge from each other within the no-imitation area. The case $b_2 = 0$ can be treated analogously. Γ is definitely negative in this case, so there is a room for convergence within the no-innovation area (it is possible that the level of imitation activity will be too low, so all these countries will grow slower than the leader and their a will eventually converge to 0). Obviously, the case $b_1 = 0$, $b_2 = 0$ is impossible in a stationary equilibrium, as far as we assume that the leader makes some innovation. ■

Proposition 2 gives a criterion of stability for the case, where the country does not imitate the highest possible technology in the stationary equilibrium. However, there is also, at least, one stationary equilibrium out of this class, namely, that of the technological leader[9]. Is it possible for a successor to catch up the leader, if both countries are identical in their ability to accumulate capital and improve technologies? The following proposition tries to answer this question.

Proposition 3 *Suppose that $f(x) = x^\alpha$. Then a stationary equilibrium with $a = 1$ is stable, if the close successors of the leader imitate its technology ($q_1(1)$ is not very high) and $q_2'(1)$ is sufficiently small* (*see inequality* (**??**) *below*)*. Otherwise, the stationary equilibrium is unstable, i. e. the successors cannot catch up with the leader.*

Proof. If $b_1 = 0$ for a close to 1, then the economy has got into the no-imitation area, and due to Proposition 2, cannot converge to the stationary equilibrium $a = 1$. Otherwise, the most advanced technology will be imitated for sufficiently high a. In this case, the innovation activity is determined by (26). Differentiating (26) at $a = 1$, we obtain

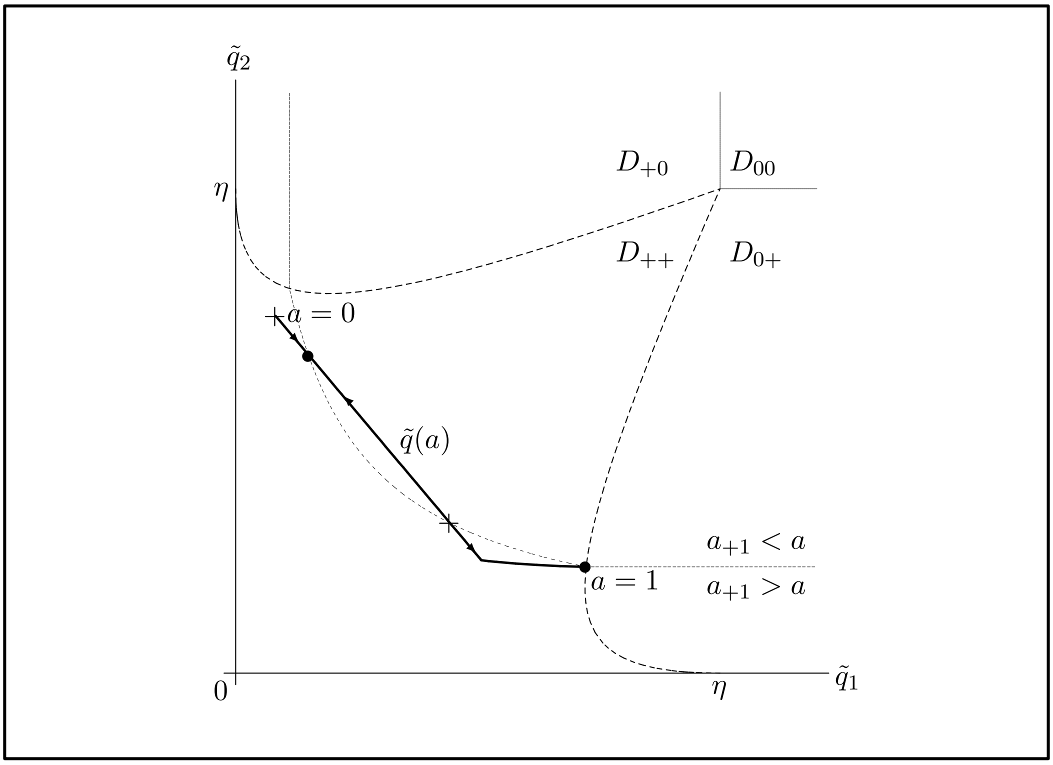
$$dv = \left(\frac{\eta\varphi_2(b_2)\left(q_1(1) - \dfrac{q_2'(1)}{q_2(1)}\right)}{q_2(1)\left(\dfrac{1}{\alpha} - \dfrac{\varphi_2'(b_2)v_2(b_2)}{\varphi_2(b_2)^2}\right)} - v_2(b_2) \right) da, \tag{41}$$

where $b_2 = \hat{b}_2(1)$. The equilibrium $a = 1$ is stable, if $dv < 0$, i. e. if $q_2'(1)$ is sufficiently low. Otherwise divergence takes place. ■

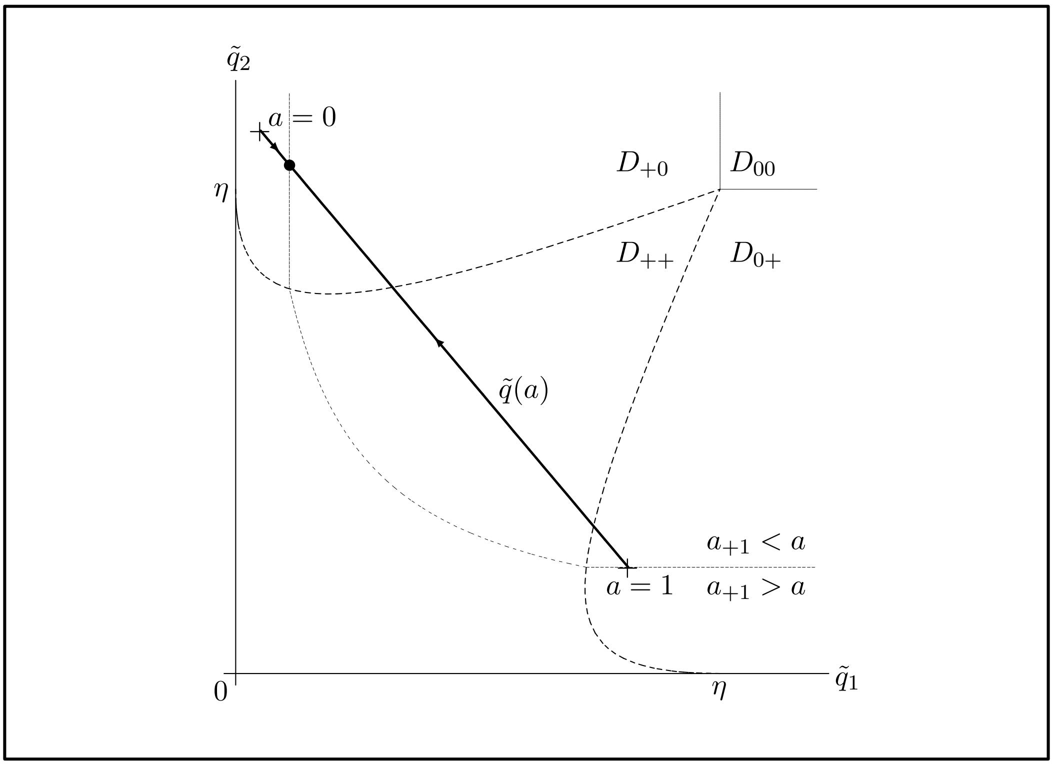
Propositions 2 and 3 show that the structure of stable and unstable stationary equilibria is generally the same as in the model without capital (see Polterovich, Tonis, 2003). Note, however, that in the model with capital, the possibility of convergence depends not only on the innovation cost function $q_2(\cdot)$, but also on the cost of imitation $q_1(1)$: higher cost implies that some capital is diverted from innovation to imitation (near the leader, the imitation activity (if any) is independent of q_1, so the imitation expenditures are increasing in q_1), which results in less active innovation. Thus, increase in $q_1(1)$ negatively affects convergence (the right-hand side of (41) is increasing in $q_1(1)$). There was no such effect in the model without capital.

Note also that Proposition 3 gives a criterion of convergence only for countries, whose country-specific exogenous parameters are identical to those of the leader. Otherwise, the point of possible convergence is not at $a = 1$, but somewhere else. For example, if the costs of innovation or imitation negatively depend on the quality of institutions, then a country, which has better institutions than the leader, is able not only to catch up, but to overtake it. On the contrary, if the quality of institutions

[9]This is really a stationary equilibrium, only if no leapfrogging takes place, i. e. when no other country occupies the leader's position. We do not consider the possibility of leapfrogging in this model, because this is not the case in reality so far, at least, in the period for which we have the data to calibrate the model.



a) Constraint (6) is binding for high a.



b) Constraint (6) is never binding.

D_{++}: $b_1 > 0$, $b_2 > 0$	D_{0+}: $b_1 = 0$, $b_2 > 0$	• stable stationary equilibrium
D_{+0}: $b_1 > 0$, $b_2 = 0$	D_{00}: $b_1 = 0$, $b_2 = 0$	+ unstable stationary equilibrium

Figure 1: Cost curve $\tilde{q}\,(a)$, steady-state curve and stationary equilibria.

for the given country is worse than for the leader, then a full catching-up is impossible, some gap will always be present. Similar effects take place for savings rate σ, if we assume that higher σ implies lower costs. Generally, a change in the savings rate or in the quality of institutions may affect the position or even structure of stationary equilibria. For example, if a country is within the area of attraction of a stationary equilibrium with low a, but close to the boundary of this area, then a short-run increase in capital accumulation may get it out of the trap. However, the investment-encouraging policy should be applied as soon as possible: after some wasted time, the economy will shift away from the boundary and leaving the trap will be much more difficult.

The analysis of the dynamics can be illustrated graphically as follows. Let us define the *cost curve* within plain (q_1, q_2) as the set of pairs $(\tilde{q}_1(a), q_2(a))$ for $a \in [0, 1]$. Suppose that $q_2(1) =$ is given and define the *steady-state curve* as the set of potential steady-state pairs (q_1, q_2), i. e. pairs for which $a_{+1}(a) = a$, where $a = \tilde{q}_1^{-1}(q_1)$ (due to Propositions 2) and 3, this definition is correct). The position of this curve depends only on $\bar{q}_2$ and does not depend on the position of the rest of the cost curve.

Now, to qualitatively analyze the dynamics, one just need to know, how the cost curve is positioned about the steady-state curve (see Figure 1a,b). The relative productivity level of the economy rises after the period if and only if point $q(a)$ lies below and/or to the left from the steady-state curve (let us call the corresponding zone "catching-up area"). Stationary equilibria correspond to intersection points of the cost curve with the steady-state curve. If the cost curve gets out of the catching-up area, as q_1 goes up (in other words, the slope of the cost curve is flatter that the slope of the steady-state curve), then the corresponding stationary equilibrium is stable; otherwise, it is unstable. For example, in Figure 1a, there is one stable steady state with $a < 1$ and one unstable[10], with $a = 1$. In Figure 1b, there are two stationary equilibria, with $a = 1$ and with low a; there is also an unstable one with intermediate a which serves as a boundary between the zones of attraction of the stable equilibria.

4 Empirical adjustment of the model

In this section, some empirical considerations will be given about the model considered above. We are going to test some of basic assumptions of the model (in particular, those concerning the per-unit cost functions of innovation and imitation) and to adjust the parameters of the model to the statistical data.

[10]The number of stationary points in dynamic systems is usually odd. Here the number is even because one of the points lies at the boundary of the phase space, which is $[0, 1]$. This equilibrium may be treated as two coincident stationary equilibria, one stable and one unstable.

We use data from two sources: the World Development Indicators (WDI, 2001-2003) and the International Country Risk Guide (ICRG, 2001)[11]. The data structure is the following:

Y_t	–	GDP per capita, PPP (constant 1995 international \$) in year $1900+t$,
		$t = 80, \ldots, 99$, 129 countries; source: WDI;
N_t	–	population in year $1900+t$,
		$t = 80, \ldots, 99$, 192 countries; source: WDI;
I_t^G	–	gross capital formation, % of GDP$1900+t$,
		$t = 80, \ldots, 99$, 180 countries; source: WDI;
I_t^N	–	net capital formation, % of GDP$1900+t$,
		$t = 80, \ldots, 99$, 136 countries; source: WDI;
R	–	quality of institutions (0.01× ICRG composite risk index),
		average over 1984–1999, 129 countries; source: ICRG;
		$R \in [0,1], R$ is higher for lower risks;
C_1	–	net royalty payments, % of GDP,
		average over 1980–1999, 120 countries; source: WDI;
C_2	–	spending on R&D, % of GDP,
		average over 1980–1999, 89 countries; source: WDI.

Using the above data, we are going to calibrate the theoretical model, i. e. estimate its parameters so as to obtain good quality of prediction (in some sense). We consider a special case of the model with Cobb-Douglas production function in the final good sector ($f(x) = x^\alpha$) and probability functions $\psi_i(b_i) = \dfrac{\mu_i}{\mu_i + b_i}$. We assume also that per-unit cost functions of innovation and imitation q_1 and q_2 depend on country-specific parameters σ and R. After having considered various specifications, we have chosen the linear representation for the cost functions:

$$q_i = q_i(a, \sigma, R) = c_{ai}a + c_{\sigma i}\sigma + c_{Ri}R + c_i. \tag{42}$$

So, we are interesting in estimating the following parameters:

α	–	parameter of the production function;
μ_1	–	maximal growth rate due to imitation;
μ_2	–	maximal growth rate due to innovation;
δ	–	capital depreciation rate;
ρ	–	technology obsolescense rate;

[11]We tried more extended specifications using Barro-Lee data on human capital, POLITY data on the quality of democracy and other ICRG risk indices. All these variables turned out to be insignificant in regressions.

T	–	duration of a time period (years);
m	–	ratio of total imitation expenditures to royalty fees (see below);
$c_{ai}, c_{\sigma i}, c_{Ri}, c_i$	–	parameters of linear cost functions $q_1(a, \sigma, R)$, $q_2(a, \sigma, R)$.

In order to estimate the parameters of the cost functions, we need proxies for b_1 and b_2. These variables are closely related to the growth rate due to innovation or imitation, so firstly we need to extract the growth rate of the "total factor productivity" from the GDP growth rate. The average per-period growth rate g of real per-capita GDP is given by

$$g = \left(\frac{Y_{99}}{Y_{80}}\right)^{T/19} - 1.$$

Due to our assumption about the Cobb-Douglas form of the production function, GDP can be represented as follows:

$$Y = A^{1-\alpha} X^{\alpha}, \tag{43}$$

where X is the amount of capital used in production and A is the productivity factor. Denote by g_X the per-period growth rate of X. According to the model, X is proportional to total capital stock K, so the growth rate of capital could be a good proxy for g_X. We have not found systematic cross-country data on capital stock or its growth rate (lack of such data can be explained by problems concerning aggregation of investment for different periods of time), so we use instead the following proxy for g_X:

$$1 + g_X = \frac{1 + \dfrac{I^N/100}{(K_t/(N_t Y_t))_{\text{av}}}}{1 + g_N}, \tag{44}$$

where I^N is the average net investment rate I_t^N, g_N is the average per-period population growth rate and $(K_t/(N_t Y_t))_{\text{av}}$ is the average capital-to-GDP ratio (all averages are taken during the period $t = 80, \ldots, 99$). The values of K_t in this ratio are calculated by the usual recurrent formula[12]

$$K_{t+1} = (1 - \delta) K_t + \sigma N_t Y_t, \tag{45}$$

where δ is to be estimated (see below) and σ is the average of $I_t^G/100$ and may be considered as a proxy for savings rate in Solow model. In order to obtain established values of K_t, we start calculating K_t from $t = 60$ (the initial value of capital stock K_{60} is chosen proportionally to $N_{60} Y_{60}$). Correction by the population growth rate in (44) is needed to take into account not only capital, but also labor in the estimation of the total factor productivity growth rate[13].

[12]Such technique known as "perpetual inventory estimation" is also used in a number of papers on growth accounting (e. g., see Bosworth and Collins, 2003 or Senhadji, 2000). The value of δ used in calculations is typically 0.04–0.05.

[13]Generally, human capital should also be taken into consideration.

To extract the total factor productivity, we use the following regression:

$$\ln(1+g) = \alpha \ln(1+g_X) + \text{const.} \tag{46}$$

From (46) we obtain the estimated value of α (it is about 0.3, higher or lower, depending on the depreciation rate δ). Now the estimation of the productivity factor A_t is determined by

$$A_t = \left(\frac{N_t Y_t}{K_t^{\alpha}}\right)^{\frac{1}{1-\alpha}}$$

(it can be seen from the data that the expenditures on imitation and innovation constitute a small part of GDP, typically, less than 2%, so, solving (43), we put $X = K$ for simplicity).

Based on A_t, we construct some other country-specific variables useful for our analysis:

$$a_t = \frac{A_t}{A_t[\text{USA}]} \quad - \quad \text{relative productivity level;}$$

$$a \quad - \quad \text{average over all } a_t,\ t = 80, \ldots, 99;$$

$$v = \frac{\left(\frac{A_{99}}{A_{80}}\right)^{\frac{T}{19}}}{(1-\rho)^T} \quad - \quad \text{proxy for productivity growth factor } v \text{ in the model.}$$

Now we are going to estimate q_1 and q_2, using the data on innovation and imitation activity. The main difficulty in our analysis is that we do not know the values of q_1 and q_2 and observe C_1 and C_2 instead. Another problem is that C_1, the share of net royalty payments in GDP does not include any costs of absorbing technologies other than royalties. Hence, C_1 is actually only a fraction of the total cost of imitation. To take this into account, let us use mC_1 instead of C_1 as a proxy for the imitation cost in our calculations. Here $m \geq 1$ is a constant to be estimated.

We need to construct some proxies for q_1 and q_2. Suppose that both C_1 and C_2 are present in the observation for the country and m is given. Let us denote

$$\zeta = \frac{mC_1}{C_2}.$$

Due to (20), we have the system of two equations with two unknown variables b_1 and b_2:

$$\begin{aligned} &(1 + b_1\psi_1(b_1))(1 + b_2\psi_2(b_2)) = v; \\ &\frac{b_1\varphi_1(b_1)(1 + b_2\psi_2(b_2))}{b_2\varphi_2(b_2)(1 + b_1\psi_1(b_1))} = \zeta, \end{aligned} \tag{47}$$

where $\psi_i(b_i) = \dfrac{\mu_i}{\mu_i + b_i}$ and $\varphi_i(b_i) = \left(\dfrac{\mu_i}{\mu_i + b_i}\right)^2$. System (47) may have multiple solutions, because the same level of expenditures may correspond to low or high level of innovation/imitation activity. However, if we require that more activity result in higher expenditures, then only one solution remains (actually, the lower one).

As far as b_1 and b_2 are calculated, q_1 and q_2 can be obtained from the first-order conditions (25).

Our next task is to estimate q_1 and q_2 as functions of phase variable a and exogenous[14] variables σ (savings rate) and R (quality of institutions), taking parameters μ_1, μ_2, δ, ρ, T and m as given. For a given setting of the parameters, coefficients c_{ai}, $c_{\sigma i}$, c_{Ri} and c_i, determining linear functions (42), can be estimated, using OLS. Thus, we have proxies for all parameters of the model.

It turns out that R is strongly positively correlated with a and (to a less extent) with σ. To avoid improper estimation results, which may occur because of multi-collinearity, we construct a new variable $\tilde{R}$:

$$\tilde{R} = R - d_a a - \sigma, \tag{48}$$

where d_a and d_σ are the parameters of the linear regression $R = R(a, \sigma)$. Thus, $\tilde{R}$ can be treated as a measure of deviation of the institutional quality from some "standard" level (for the given a and σ). This $\tilde{R}$ will be used instead of R in our estimation of the cost functions[15].

Now, given parameters μ_1, μ_2, δ, ρ, T and m and estimated linear cost functions $q_1(a, \sigma, \tilde{R}), q_2(a, \sigma, \tilde{R})$, we can study the evolution of the model world and compare it to that of the real world. We have a sample of 83 countries, for which all necessary data are present. Each country starts from the known pair (k_80, a_80). First-order conditions (19) along with balance equation (24) determine innovation-imitation policy $(b_1, b_2) = (\hat{b}_1(a, \sigma, \tilde{R}), b_2(a, \sigma, \tilde{R}))$ (these are the predicted values of the imitation and innovation growth rates; they do not coincide with the proxies for b_1 and b_2). Now we just use formulas (31) and (32) to obtain k_{+1} and a_{+1} as functions of k and a. In particular, a_{+1} is determined as follows:

$$a_{+1}(a, \sigma, \tilde{R}) = \frac{(1 + \hat{b}_1(a, \sigma, \tilde{R})\psi_1(\hat{b}_1(a, \sigma, \tilde{R})) + \hat{b}_2(a, \sigma, \tilde{R})\psi_2(\hat{b}_2(a, \sigma, \tilde{R})))}{1 + \hat{b}_2(1, \sigma[\text{USA}], \tilde{R}[\text{USA}])\psi_2(\hat{b}_2(1, \sigma[\text{USA}], \tilde{R}[\text{USA}]))} a. \tag{49}$$

In order to check, whether the model is consistent with the reality, let us try to predict the relative productivity level of countries in 1999, given that in 1980:

$$\hat{a}_{99} = a_{+1}(a_{+1}(\ldots a_{+1}(a_{80}, \sigma, \tilde{R}) \ldots, \sigma, \tilde{R}), \sigma, \tilde{R}) \qquad \text{(19/T times)}.$$

In a similar way we can build a prediction for k_{99}. Now the predicted relative GDP level $\hat{y}_{99}$ can be calculated as follows:

$$\hat{y}_{99} = \frac{Y_{99}}{Y_{99}[\text{USA}]} = a_{99} \frac{\tilde{y}_{99}}{\tilde{y}_{99}[\text{USA}]},$$

[14]Although, these variables are endogenous to some extent: in particular, there is evidence that institutional quality may depend on the level of economic development.

[15]This approach may also partially solve the problem of endogenous institutional quality, provided that the relative level of development mostly affects the trend rather than the deviation of R.

where $\tilde{y}_t = v_t \left(\dfrac{k_t}{v_t + \eta z_t} \right)^\alpha$.

Now we can choose μ_1, μ_2, δ, ρ, T and m so as to achieve the best prediction. There are many possible ways to measure the quality of prediction. Let us consider the following criteria:

Minimal sum of squares: the sum of squares of logarithmic errors

$$\sum (\ln \hat{y}_{99} - \ln y_{99})^2$$

is minimized, where y_{99} and $\hat{y}_{99}$ are, respectively, actual and predicted relative GDP values. The objective function is unimodal, so it is easy to find its minimum. However, this criterion is sensitive to outliers.

Coincidence of directions: the predicted direction of change in y during the period 1980–1999 must coincide with the actual one for as many countries as possible. This criterion does not take into account the speed of changes. It is more robust to outliers than the previous one. However, it is algorithmically difficult to solve the optimization problem because the objective function turns out to be multi-modal, so one cannot be sure that the obtained local optimum is also global.

Coincidence of distributions: parameters are chosen so that the density function of distribution of $\ln \hat{a}_{99}$ be close to that of $\ln a_{99}$ (according to the least squares criterion). This criterion is useful for dealing with distribution densities rather than detailed cross-country data. Since the estimated density functions are defined within a discrete domain and have a discrete set of possible values, the problem of multi-modality is also present for this criterion.

According to theses criteria, the best-predictive values of the parameters are the following:

Minimal sum of squares:

$\alpha = 0.285, \quad \eta = 2.509;$

$\mu_1 = 0.964 \quad (23.5\%$ per year);

$\mu_2 = 1.574 \quad (34.3\%$ per year);

$\delta = 0.153 \quad (5.04\%$ per year);

$\rho = 0.367 \quad (13.3\%$ per year);

$T = 3.20$ years;

$m = 3.05.$

Estimated cost functions:

$$q_1(a, \sigma, \tilde{R}) = \underset{(2.57)}{0.477} a - \underset{(-2.36)}{2.750}\, \sigma - \underset{(-1.66)}{1.941}\, \tilde{R} + \underset{(4.98)}{1.314} \quad (R^2 = 0.241);$$

$$q_2(a) = -\underset{(-2.31)}{0.483}\, a + \underset{(10.8)}{1.003} \quad (R^2 = 0.106).$$

Standard error of prediction of $\ln y$: 0.227;

Percentage of incorrectly predicted directions: 16.1%;

Standard error of prediction of density: 0.0450.

Coincidence of directions:

$\alpha = 0.295, \quad \eta = 2.391;$

$\mu_1 = 0.596 \quad (18.9\% \text{ per year});$

$\mu_2 = 0.827 \quad (25.0\% \text{ per year});$

$\delta = 0.122 \quad (4.71\% \text{ per year});$

$\rho = 0.243 \quad (9.80\% \text{ per year});$

$T = 2.70$ years;

$m = 3.14.$

Estimated cost functions:

$$q_1(a, \sigma, \tilde{R}) = \underset{(2.44)}{0.483} a - \underset{(-2.50)}{3.097}\, \sigma - \underset{(-1.74)}{2.156}\, \tilde{R} + \underset{(5.56)}{1.553} \quad (R^2 = 0.245);$$

$$q_2(a) = -\underset{(-2.41)}{0.499}\, a + \underset{(12.1)}{1.108} \quad (R^2 = 0.199).$$

Standard error of prediction of $\ln y$: 0.251;

Percentage of incorrectly predicted directions: 9.7%;

Standard error of prediction of density: 0.0807.

Coincidence of distributions:

$\alpha = 0.280, \quad \eta = 2.567;$

$\mu_1 = 0.395 \quad (19.1\% \text{ per year});$

$\mu_2 = 0.589 \quad (27.6\% \text{ per year});$

$\delta = 0.096 \quad (5.20\% \text{ per year});$

$\rho = 0.215 \quad (11.96\% \text{ per year});$

$T = 1.90$ years;

$m = 2.80.$

Estimated cost functions:

$$q_1(a, \sigma, \tilde{R}) = \underset{(2.57)}{0.573} a - \underset{(-2.65)}{3.711}\, \sigma - \underset{(-1.58)}{2.220}\, \tilde{R} + \underset{(5.36)}{1.699} \quad (R^2 = 0.389);$$

$$q_2(a) = -\underset{(-2.38)}{0.620}\, a + \underset{(10.2)}{1.178} \quad (R^2 = 0.112).$$

Standard error of prediction of $\ln y$: 0.228;

Percentage of incorrectly predicted directions: 16.1%;

Standard error of prediction of density: 0.0243.

Here t-statistics are given below the coefficients, in parentheses. Variables σ and $\tilde{R}$ are not included in the estimation of the innovation cost function q_2, because they are insignificant in the corresponding regressions.

Thus, our hypotheses about the cost functions q_1 and q_2 are proved empirically: q_1 is increasing in a and q_2 is decreasing in a with high level of significance. Note also that the savings rate and the institutional quality negatively affect the cost function of imitation (with 7–10% level of significance) and do not significantly affect the cost function of innovation. One of possible explanations of this phenomenon is that the data we use mostly concern the expenditures on local innovations. Only a small part of these R&D expenditures is immediately materialized in large investment projects involving foreign capital, so this activity does not necessarily require good institutions. On the contrary, the process of imitation usually requires the interaction with foreign investors, which is sensitive to the institutional climate.

Alternatively, one could consider a model without capital, when the resource needed for production of intermediate goods and investment in technology can be bought by fixed price 1 (as in Polterovich, Tonis, 2003). If regressions for q_1 include only a and R (as before, q_2 significantly depends on a only), then we obtain the following results (each one is best with respect to the corresponding criterion):

Standard error of prediction of $\ln y$: 0.300 (comparing to 0.227 in the model with capital);

Percentage of incorrectly predicted directions: 22.6% (comparing to 9.7%);

Standard error of prediction of density: 0.0837 (comparing to 0.0243).

If savings rate σ is included in the regression for q_1, then the quality of prediction is characterized as follows:

Standard error of prediction of $\ln y$: 0.284;

Percentage of incorrectly predicted directions: 22.6%;

Standard error of prediction of density: 0.0617.

Thus, taking into account investment decisions and capital accumulation improves the quality of approximation.

The evolution of the distribution of $\ln y$ is depicted in Figure 2. One can see that both in 1980 and in 1999, the distribution density has two local peaks, and during the period, the gap between the peaks gets deeper and wider. The right peak ($\ln y \approx -0.4$) consists of a group of intensively growing countries with relatively high quality of institutions (Norway, Ireland, Finland) or high investment rate

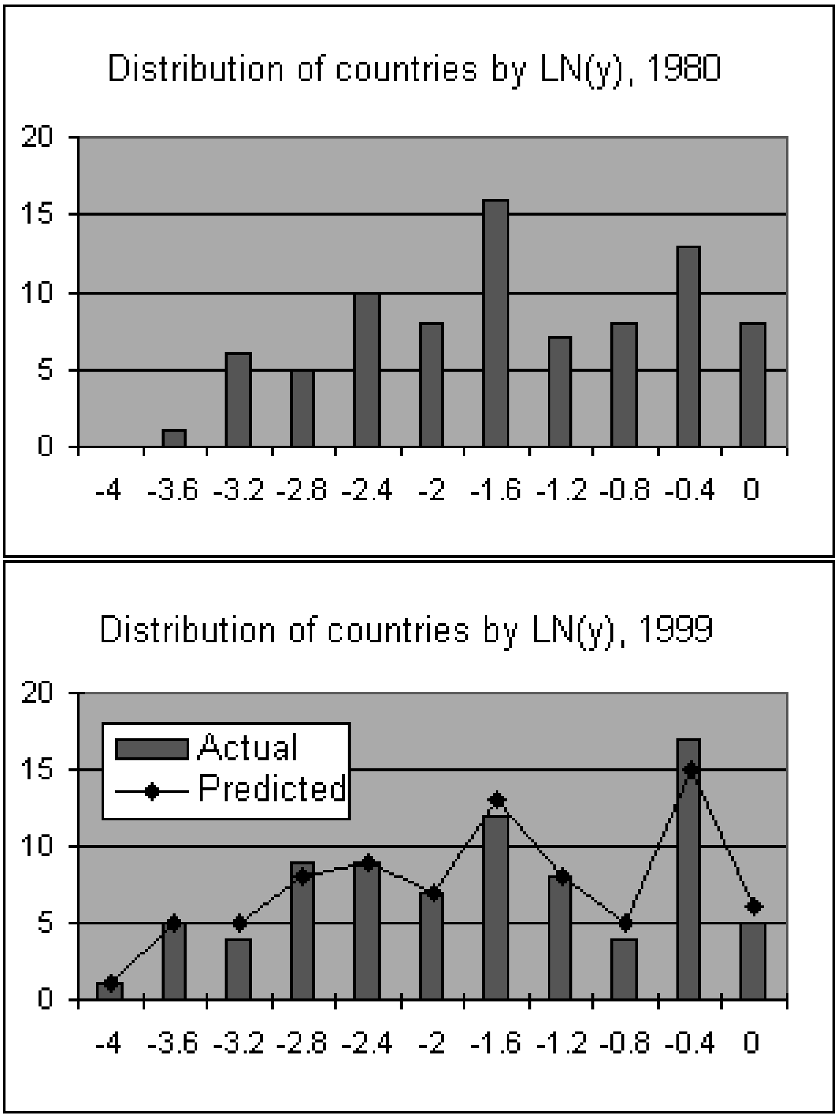


Figure 2: Distribution of countries with respect to $\ln y$

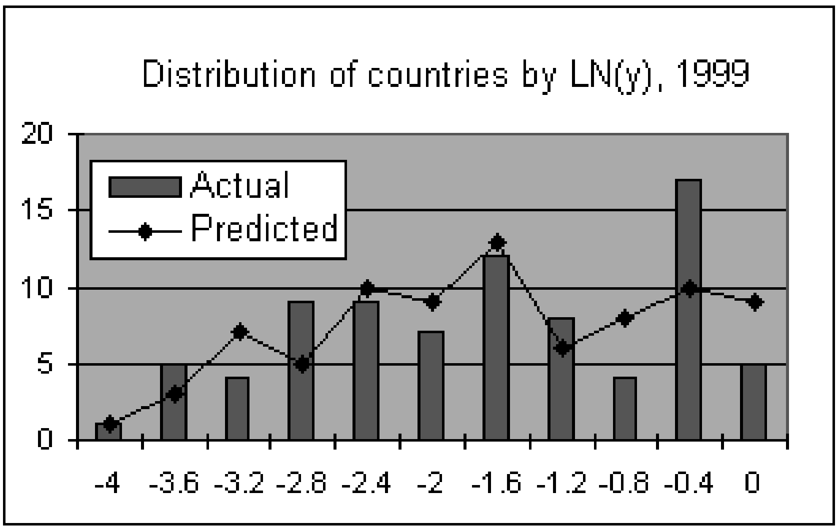


Figure 3: Model without capital fails to predict the right peak.

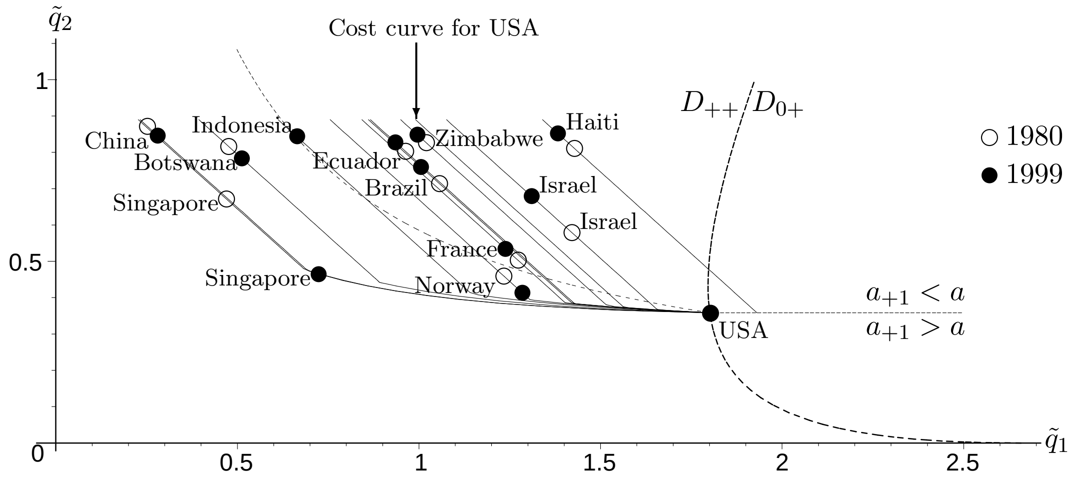


Figure 4: Cost curves for various savings rates and institutional levels.

(Hong Kong) or both (Japan, Singapore). These countries have a real opportunity to approach the leader (USA). However, within the same peak, there are some other countries with weaker institutions and decreasing a: Belgium, Austria, France, Italy. They are gradually moving away from the frontier. The left peak is getting distributed around a wide range, rather in the left part of the picture and consists mostly of countries moving backward. It is likely that these countries develop at a steadily lower growth rate than the leaders and their y is converging to zero. The above theoretical analysis suggests that these peaks may correspond to stable fixed points of $a_{+1}(a, \sigma, \tilde{R})$ for the most typical levels of σ and $\tilde{R}$.

Figure 3 depicts the actual and predicted distribution density for the model without capital (savings rate σ is included in the regression for q_1). One can see that this model does not predict the formation of the right peak. Comparing Figure 2 with Figure3 shows the extent to which taking into account capital accumulation improves the quality of prediction.

Figure 4 depicts typical cases of positioning the estimated cost curve with respect to the steady-state curve. If the savings rate and the quality of institutions are low (Haiti), then all of the cost curve lies above the steady-state curve, so the country will gradually fall to the lowest possible productivity level ($a = 0$), no matter where it has started. In this case, $a = 0$ is a unique stable stationary equilibrium. The corresponding point $(\tilde{q}_1, \tilde{q}_2)$ lies relatively close to the pure imitation area D_{+0}. There is also an unstable equilibrium $a = 1$.

For intermediate σ and/or $\tilde{R}$, the equilibrium $a = 1$ becomes stable and a new unstable equilibrium with $a < 1$ occurs. Countries with high a (Norway) approach the leader (USA), whereas countries with low a (Brazil, Ecuador, France, Israel, Zimbabwe) move away from the leader, towards the trap equilibrium $a = 0$. One can see from Figure 4 that a follower country with the same country-specific parameters as USA is able to catch up with the leader, only if it has started from very high a.

For even higher σ and $\tilde{R}$ (Indonesia), zero equilibrium becomes unstable and a new stable stationary equilibrium with positive a occurs. There are also tho stationary equilibria: $a = 1$ (stable) and $a < 1$ (unstable). One can see that Indonesia is near the lower stable equilibrium and its position remains almost unchanged from 1980 to 1999.

Finally, countries with the highest high σ and $\tilde{R}$ (Botswana, China, Singapore) have all of their cost curve lying within the catching-up area, so these countries are intensively growing and are expected to catch up with the leader eventually, no matter where they have started from.

5 Conclusion

The results described above support empirical findings that the convergence problem, a central problem of the economic growth theory, may be better understood in the framework of imitation-innovation models, and that quality of institutions has to be taken into account. We show how and why the capital stock accumulation may not influence qualitatively the asymptotic club behavior. Our calibration results make it plausible, however, that savings rates may have impact on asymptotics, and this question merits to be studied in greater detail.

Empirical studies seem to show that imitations and innovations are rather complementary than substitutable (see Polterovich and Popov (2003, Stages of Development and Economic Growth. Manuscript)). This was the case in the model without capital. However, in the new version of the model, this fact takes no place. Another important divergence with the reality: the most intensive innovators, including USA, imitate a lot. Thus, one has to take into account that a part of followers' innovations is not local and may be borrowed. This part increases when the country level of development gets higher.

It is quite plausible that the cost of imitation depends not only on the distance to frontier but also on the position of a follower among other countries. This line of generalization leads us to a class of models that were started to study by Henkin and Polterovich (1999).

We treat the institutional quality as exogenously given. It is well known, however, that the dependence is two-sided (Chong, Calderon (2000)). Taking this into account may change the structure of the asymptotic behavior so that several mixed (imitation plus innovation) equilibria arise.

There is no doubt now that high inequality supported by the club convergence is harmful for worldwide growth. One could ask how the total world wealth might be redistributed to increase growth rates and consumption levels of all countries. This is an important topic for future research.

References

Acemoglu, D., Ph. Aghion, and F. Zilibotti (2002a). Distance to Frontier, Selection, and Economic Growth. June 25, 2002
(http://post.economics.harvard.edu/faculty/aghion/papers/Distance_to_Frontier.pdf)

Acemoglu, D., Ph. Aghion and F. Zilibotti (2002b). Vertical Integration and Distance to Frontier. August 2002
(http://post.economics.harvard.edu/faculty/aghion/papers/vertical_integration.pdf)

Aghion, Ph., and P.Howitt (1998). Endogenous Growth Theory. The MIT Press, Cambridge, Massachusetts, p694.

Azariadis, C., and A. Drazen, (1990). Threshold Externalities in Economic Development. Quarterly Journal of Economics, 105(2): 501–526.

Barro, R. J., and X. Sala-i-Martin (1995). Economic Growth. New York: McGraw-Hill.

Barro, R. J. (1996). Institutions and Growth: An Introductory Essay. Journal of Economic Growth, 1(1): 145–148.

Bosworth, B. and S. Collins (2003). The Empirics of Growth: An Update. Brookings Institution, September 22, 2003.

Bresis, E., P. Krugman, and D. Tsiddon (1993). Leapfrogging in international Competition: A Theory of Cycles in National Technological Leadership, American Economic Review, V. 83, No. 5. 1211–1219.

Chong, A., and C. Calderon (2000). Causality and Feedback Between Institutional Measures and Economic Growth. Economics and Politics, v. 12.

Easterly W., and R.Levine (2000). It's not factor accumulation: stylized facts and growth models.

Feyrer, J. (2003). Convergence by Parts. "http://www.dartmouth.edu/~jfeyrer/parts.pdf"

Henkin, G., and V.Polterovich (1999). A Difference-Differential Analogue of the Burgers Equation and Some Models of Economic Development. Discrete and Continuous Dynamic Systems, 5(4): 697–728.

Howitt, P. and D. Mayer-Foulkes (2002). R&D, Implementation And Stagnation: A Schumpeterian Theory Of Convergence Clubs. NBER Working Paper 9104.

Iwai, K. (1984). Schumpeterian Dynamics, Part I: An evolutionary model of innovation and imitation. Journal of Economic Behavior and Organization, v. 5, 159–190.

Iwai, K. (1984). Schumpeterian Dynamics, Part II: Technological Progress, Form growth and "Economic Selection". Journal of Economic Behavior and Organization, v. 5, 287–320.

Pack, H. (2001).Technological Change and Growth in East Asia: Macro Versus Micro Perspectives. In Stiglitz J. E. and S. Yusuf (eds.): Rethinking the East Asian Miracle. Ch. 3, 95–142.

Polterovich, V., G. Henkin (1988). An Evolutionary Model of the Interaction of the Processes of Creation and Adoption of Technologies. Economics and Mathematical Methods, v. 24, N. 6, 1071–1083 (in Russian).

Polterovich, V., A. Tonis (2003). Innovation and Imitation at Various Stages of Development. Unpublished paper presented at NES research conference in October, 2003.

Quah, D.T. Empirical Cross- Section Dynamics in Economic Growth. European Economic Review, 37, 426-34.

Ruttan, V. (1997). Induced Innovation, Evolutionary Theory, and Path Dependence: Sources of Technical Change. Economic Journal, V. 107, September, 1520–1529.

Segerstrom P. S. (1991). Innovation, Imitation, and Economic Growth. Journal of Political Economy, v. 99, N. 4, 807–827.

Semenov, A.S. (2004) The innovations In The Economy With The Exhaustible Resource Sector. Working Paper #BSP/2004/073. - Moscow, New Economic School, 2004, 34pp. (Rus.).

Senhadji, A. (2000). Sources Of Economic Growth: An Extensive Growth Accounting Exercise. IMF Staff Papers, Vol. 47, No. 1, pp. 129–157.

Silverberg G. and B. Verspagen (1994). Economic Dynamics and Behavior Adaptation: An Application To An Evolutionary Endogenous Growth Model. IIASA Working Paper, WP–94–84, Sep.